Comparison of Pricing Approaches for Longevity Markets

Melvern Leung

Monash University

melvern.leung@monash.edu

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Background

- Longevity Risk in Pensions and annuity providers.
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- LAGIC in Australia vs Solvency II in U.K.
Background

- Longevity Risk in Pensions and annuity providers.
- LAGIC in Australia vs Solvency II in U.K.
- Longevity linked securities: Bonds, swaps, options.
Private sector wants to diversify their portfolio, annuity providers/pension funds wants to hedge their longevity risk. Win-Win.
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Create products which gives incentive for private sector to buy into.
Motivation

- Private sector wants to diversify their portfolio, annuity providers/pension funds wants to hedge their longevity risk. Win-Win.
- Create products which gives incentive for private sector to buy into.
- Finding a "fair price".
Mortality modeling and forecasting under the CBD-model with a state-space representation.
Setup

- Mortality modeling and forecasting under the CBD-model with a state-space representation.

- Discuss each of the four approaches used to price an $s$-forward
Mortality modeling and forecasting under the CBD-model with a state-space representation.

Discuss each of the four approaches used to price an s-forward

Compare the results obtained.
Cairns Blake and Dowd Model

Denote a 1 year death probability for a person currently aged $x$ at time $t$ by $q_{x,t}$, then this can be modeled via,

$$\ln \left( \frac{q_{x,t}}{1 - q_{x,t}} \right) = \kappa_{1,t} + \kappa_{2,t}(x - \bar{x}).$$  \hspace{1cm} (1)

We adapt this model to incorporate an error component in the measurement equation so that a state-space approach can be applied.

$$\ln \left( \frac{q_{x,t}}{1 - q_{x,t}} \right) = \kappa_{1,t} + \kappa_{2,t}(x - \bar{x}) + \varepsilon_t$$  \hspace{1cm} (2)

Cairns et al. (2006) suggests that $\kappa_{1,t}$ and $\kappa_{2,t}$ can be modeled by a 2-Dimension random walk with drift,

$$\begin{bmatrix} \kappa_{1,t} \\ \kappa_{2,t} \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} \kappa_{1,t-1} \\ \kappa_{2,t-1} \end{bmatrix} + \omega_t$$
Statespace framework

Our framework is as follows:

\[
y_t = \ln \left( \frac{1 q_{x,t}}{1 - 1 q_{x,t}} \right) = \begin{bmatrix} 1 & (x_1 - \bar{x}) \\ 1 & (x_2 - \bar{x}) \\ \vdots & \vdots \\ 1 & (x_n - \bar{x}) \end{bmatrix} \begin{bmatrix} \kappa_{1,t} \\ \kappa_{2,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \vdots \end{bmatrix}, \tag{3}
\]

\[
\begin{bmatrix} \kappa_{1,t} \\ \kappa_{2,t} \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} \kappa_{1,t-1} \\ \kappa_{2,t-1} \end{bmatrix} + \omega_t. \tag{4}
\]

Where \( \varepsilon_t \sim \text{i.i.d } N(0, \sigma^2_{\varepsilon}) \) and \( \omega_t \sim N(0, \Sigma_\omega) \).
Prior Choices

\[
\begin{align*}
\pi(\sigma^2_\varepsilon) & \sim I.G(a_\varepsilon, b_\varepsilon) \\
\pi(\theta) & \sim N(\mu_\theta, \Sigma_\theta) \\
\pi(\Sigma_\omega | \Sigma_{11}, \Sigma_{22}) & \sim I.W \left(\nu + 2 - 1, 2\nu \text{ diag} \left(\frac{1}{\Sigma_{11}}, \frac{1}{\Sigma_{22}}\right)\right) \\
\pi(\Sigma_{kk}) & i.i.d. \sim I.G \left(\frac{1}{2}, \frac{1}{A_k}\right) \text{ for } k \in (1, 2)
\end{align*}
\]

Priors where chosen such that they had conjugate forms to their respective likelihoods. A hierarchical structure was chosen for \(\Sigma_\omega\) was due to the biased caused from a regular Inverse-Wishart prior (Huang et al., 2013; Gelman et al., 2006). The hyper parameters were chosen such that the priors were non-informative.
Summary Statistics

N=10000 draws, 3000 burn-in period, Australian dataset 1961-2011.

**Table 1:** Summary Statistics for parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Posterior Mean</th>
<th>95% HPD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1$</td>
<td>0.004793253</td>
<td>$(-0.02485228, -0.005813835)$</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>0.000422871</td>
<td>$(0.000072951168, 0.0007697948)$</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>0.00263008</td>
<td>$(0.002458405, 0.002815683)$</td>
</tr>
<tr>
<td>$\Sigma_{11}$</td>
<td>0.001156807</td>
<td>$(0.0007116917, 0.001823677)$</td>
</tr>
<tr>
<td>$\Sigma_{12}$</td>
<td>$2.425724e-05$</td>
<td>$(1.084455e-05, 4.249324e-05)$</td>
</tr>
<tr>
<td>$\Sigma_{22}$</td>
<td>$1.56558e-06$</td>
<td>$(9.0352e-07, 2.516597e-06)$</td>
</tr>
</tbody>
</table>
Fitted curves $\kappa_t$
35-step ahead cohort direction forecast curve

Let $k$ denote the $k^{th}$ step ahead forecast, and $N$ be the draws after the burn in period and $n = 1, \ldots, N$. Then, 

$$
\kappa_{T+k}^n \sim N(\kappa_{T+k-1}^n + \theta^n, (\Sigma_\omega)^n),
$$

$$
y_{T+k}^n \sim N(\kappa_{T+k,1} + (x - \bar{x})\kappa_{T+k,2}, (\sigma_\varepsilon^2)^n)\).\)
We look at 4 different pricing approaches:

1) Risk-neutral method (Cairns et al., 2006)
2) The 2-factor Wang transform (Wang, 2002)
3) Canonical valuation/ Maximum entropy method (Li and Ng, 2011)
4) An economic approach/ Tatonnement economics (Zhou et al., 2015)

The first two of these methods require data to find the risk-premium $\lambda$. Hence, we will use the issued but not sold EIB-bond to calibrate.
Using the setup for the EIB-bond, we apply Australian mortality projections to males aged 65 with a longevity spread of $\delta = 0.002$ over a $T = 25$ year period.

1) The price issued by EIB/BNP was in 2004, we assume that the prices have not been inflated since that time for 2011.

2) The setup was for England and Welsh males aged 65, we assume that the longevity spread $\delta$ would be kept the same for a population in Australia.

3) For ease of calculations, we assume a constant interest rate of 3%.

$$\bar{\Pi}_t(x, T) = \sum_{i=1}^{T} P(t, i) e^{\delta i} \hat{S}(x, i)$$

Under these assumptions, we find that the bond price $\bar{\Pi}_0 \approx 13.46739$
s-forward

**Definition**

An *s*-forward contract is a swap where the fixed rate payer pays an amount $K \in (0, 1)$ in exchange for the realised survival probability $T_p_x$. An S-forward contract for a population aged $x$, over a maturity period $T$, will thus have a pricing formula given by:

$$SF(x, t, T, K) = P(t, T)E_Q [T_p_x - K|F_t].$$

Since an S-Forward contract has $0$ inception cost, we have to find the value of $K(T)$ such that there will be no upfront cost.

$$K(T) = E_Q [T_p_x | F_t]$$
Pricing an $s$-forward

Under the 4 different pricing methodologies, if we were to price an $s$-forward, then the chosen $K(T)$ will be as follows:

1) Under Risk-neutral pricing method,

$$K(T) = E_Q [T p_x | \mathcal{F}_t] = \tilde{S}(x, T)$$
Under the 4 different pricing methodologies, if we were to price an s-forward, then the chosen $K(T)$ will be as follows:

1) Under Risk-neutral pricing method,
$$K(T) = E_Q \left[ \tau p_x | \mathcal{F}_t \right] = \tilde{S}(x, T)$$

2) Under Wang Transform Method,
$$K(T) = E \left[ Q \left( \Phi^{-1}(S(x, t)) + \lambda \right) \right]$$
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3) Under Canonical Valuation method,
   
   $$K(T) = \tau p_x^{market}$$
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Under the 4 different pricing methodologies, if we were to price an $s$-forward, then the chosen $K(T)$ will be as follows:

1) Under Risk-neutral pricing method,
   
   \[ K(T) = E_Q [T_p^x | F_t] = \tilde{S}(x, T) \]

2) Under Wang Transform Method,
   
   \[ K(T) = E \left[ Q \left( \Phi^{-1}(S(x, t)) + \lambda \right) \right] \]

3) Under Canonical Valuation method, $K(T) = T_p^x \text{market}$

4) Under the Tatonnement Approach, the value $K(T)$ is determined by the market under no risk-neutral assumptions. Hence, the value $K(T)$ will be found via an algorithm.
Risk-neutral pricing method

Our 2-D random walk with drift process:

$$\kappa_t = \theta + \kappa_{t-1} + (\Sigma_\omega)^{\frac{1}{2}}Z$$

Where, $$(\Sigma_\omega)^{\frac{1}{2}}(\Sigma_\omega)^{\frac{1}{2}} = \Sigma_\omega$$ and $Z \sim N(0, I)$ is under real-world probability measure $\mathbb{P}$. Cairns et al. (2006) suggested that similar to the continuous time case, we can convert to the risk-neutral density (equivalent martingale measure) by,

$$\tilde{Z} = \lambda + Z,$$

Where, $\lambda$ is the market price of longevity risk. Then,

$$\kappa_t = \kappa_{t-1} + (\theta - (\Sigma_\omega)^{\frac{1}{2}}\lambda) + (\Sigma_\omega)^{\frac{1}{2}}\tilde{Z}$$
Risk-neutral pricing method

Under risk-neutral assumption the EIB-bond price is given by:

$$\tilde{\Pi}_t(x, T, \lambda) = \sum_{i=1}^{T} P(t, i)E_{Q(\lambda)} \left[ e^{-\int_{t}^{i} \mu(x(u))du} | \mathcal{F}_t \right].$$

Matching the price at initial time $t = 0$,

$$\sum_{i=1}^{T} P(0, i)e^{\delta i} S(65, i) = \sum_{i=1}^{T} P(0, i)\tilde{S}(65, i, \lambda)$$

<table>
<thead>
<tr>
<th>Market Price of Risk</th>
<th>Value</th>
<th>$\tilde{\Pi}_0(65, 25)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\lambda_1, \lambda_2)$</td>
<td>(0.27307, 0.27307)</td>
<td>13.46739</td>
</tr>
<tr>
<td>$(\lambda_1, \lambda_2)$</td>
<td>(0.24505, 0)</td>
<td>13.46739</td>
</tr>
</tbody>
</table>
2-factor Wang Transform

Wang (2002) gave a universal pricing method, such that, assume we have a liability $X$ over a time period $[0, T]$ with $F_X(x) = P(X < x)$, then with a market price of risk $\lambda$, the risk-adjusted (distorted) function of $F(X)$ can be found by,

$$F^*(x) = Q\left(\Phi^{-1}(F(x)) + \lambda\right)$$

Where, $F^*(x)$ is the risk-adjusted function for $F(x)$. Since our aim is to find longevity risk, we have,

$$\tilde{S}(x, t) = E\left[Q\left(\Phi^{-1}(S(x, t)) + \lambda\right)\right] \text{ for } t \in [0, T]$$

Where $Q \sim \text{Student} - t(\nu)$. 
2-factor Wang Transform

To find $\lambda$ using the EIB-bond

$$\tilde{\Pi}_t(x, T, \lambda) = \sum_{i=1}^{T} P(t, i) Q \left( \Phi^{-1}(S(x, T)) + \lambda \right).$$  \hspace{1cm} (6)

Matching the price at initial time $t = 0$,

$$\sum_{i=1}^{T} P(0, i)e^{\delta i} S(65, i) = \sum_{i=1}^{T} P(0, i)\tilde{S}(65, T)$$

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<th>Market Price of Risk Value</th>
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</tr>
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<tbody>
<tr>
<td>$\lambda$</td>
<td>0.3478043</td>
</tr>
</tbody>
</table>
The maximum entropy principle was first proposed by Stutzer (1996) and used by Kogure and Kurachi (2010); Foster and Whiteman (2006) to find market survival probability denoted by \( T p_x^{\text{market}} \). In our case we by using the EIB-Bond in combination with the maximum entropy principle to find \( T p_x^{\text{market}} \).
Canonical Valuation methodology

1. \( p_j^x = (1p_j^x, 2p_j^x, ..., Tp_j^x) \), for \( j = 1, ..., N \). and let \( \pi \) denote the empirical distribution for \( p_x \).

2. \( \bar{\Pi} \) denote the market price of the EIB-bond \( \bar{\Pi}(65, 25) \).

3. Let \( \pi^* \) be the risk-neutral distribution for \( \pi \), then \( \sum_{j=1}^{N} \Pi^j \pi_j^* = \bar{\Pi} \).

4. Then the maximum entropy principle stipulates that, \( \pi^* \) should minimize the Kullback-Leibler Information divergence, \( \sum_{j=1}^{N} \pi_j^* \ln \left( \frac{\pi_j^*}{\pi_j} \right) \), subject to the constraint \( \pi_j^* > 0 \) and \( \sum_{j=1}^{N} \pi_j^* = 1 \).
Kapur and Kesavan (1992) derived the solution to the minimization of \( \sum_{j=1}^{N} \pi_j^* \ln \left( \frac{\pi_j^*}{\pi_j} \right) \) subject to \( \bar{\Pi} \), is given by

\[
\hat{\pi}_j^* = \frac{\pi_j \exp(\gamma \bar{\Pi}^j)}{\sum_{j=1}^{N} \pi_j \exp(\gamma \bar{\Pi}^j)}
\]

2. Find \( \gamma \) via, \( \bar{\Pi} = \frac{\sum_{j=1}^{N} \bar{\Pi}^j \exp(\gamma \bar{\Pi}^j)}{\sum_{j=1}^{N} \exp(\gamma \bar{\Pi}^j)} \).

3. \( \sum_{j=1}^{N} t p_x^j \pi_j^* = t p_x^{market} \)

In this scenario, we don’t assume there is a risk premium \( \lambda \), but there is a \( \gamma \) parameter which ”corrects” the real world probability \( t p_x \) to adjust for the market accepted \( T p_x^{market} \).
Tatonnement Approach

This was first suggested by (Zhou et al., 2015). In summary, we are trying to find an accepted strike price $K$, that is accepted by the market.

1. Assume we have a buyer (investor) of an $s$-forward and a seller (hedger).
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2. Denote the following parameters:

\[
\theta^A = \sup \theta^A \mathbb{E} \left[ U \left( \omega^A_t - 1 - e^{-r} - \theta^A g(S(x,t)) - f(S(x,t)) \right) \right]
\]

\[
\theta^B = \sup \theta^A \mathbb{E} \left[ U \left( \omega^B_t - 1 + e^{-r} + \theta^B g(S(x,t)) \right) \right]
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### Tatonnement Approach

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   - $\omega^A_t$ is the wealth of A, $\omega^B_t$ is the wealth of B at time $t$.
   - Let the function $g(S(x, t))$ denote the gains of the $s$-forward at time $t$. 

\[
\begin{align*}
\theta_A &= \sup \theta_A \mathbb{E}\left[U\{\omega^A_t - 1 - e^{r - \theta_A g(S(x, t)) - f(S(x, t))}\}\right] \\
\theta_B &= \sup \theta_B \mathbb{E}\left[U\{\omega^B_t - 1 + e^{r + \theta_B g(S(x, t))}\}\right]
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   - $\omega_t^A$ is the wealth of A, $\omega_t^B$ is the wealth of B at time $t$.
   - Let the function $g(S(x, t))$ denote the gains of the $s$-forward at time $t$.
   - Let $f(S(x, t))$ represent the payout for the survival probability for the hedger at time $t$. 
Tattonnement Approach

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3. $\theta_A = \sup_{\theta_A} E \left[ U\{\omega_{t-1}^A e^r - \theta^A g(S(x, t)) - f(S(x, t))\} \right]$
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4. $\theta_B = \sup_{\theta_A} E \left[ U\{\omega_{t-1}^B e^r + \theta^B g(S(x,t))\} \right]$
Tatonnement Approach

Since $g$ is an $s$-forward, $g = (S - K)$, and choosing an Exponential Utility function, we use the algorithm suggested by (Zhou et al., 2015).

for each time period $t \in [1, T]$:

1. Guess an initial $K$. 

---

The 4 Pricing methodologies

s-forward
Tattonnement Approach

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for each time period $t \in [1, T]$

1. Guess an initial $K$.
2. Determine $\theta_A$ and $\theta_B$. 

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1. Guess an initial $K$.
2. Determine $\theta_A$ and $\theta_B$.
3. if $\theta_A = \theta_B$ then stop, and set $K(t) = K$
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for each time period \( t \in [1, T] \)

1. Guess an initial \( K \).
2. Determine \( \theta_A \) and \( \theta_B \).
3. if \( \theta_A = \theta_B \) then stop, and set \( K(t) = K \)
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   1) \( K^{i+1} = K^i + h^i \)
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4. else, update $K$ by:
   1) $K^{i+1} = K^i + h^i$
   2) $h^i = \gamma |K^i| (\theta_B - \theta_A)$
Results

For each of the methods, 1000 chains from the MCMC was used after burn-in. A monte-carlo average was taken when an expectation was involved. Using a portfolio of aged 65, with a hedging period of $T = 5, 10, 15, 20, 25$ of the $s$-forward. The prices are shown below:

<table>
<thead>
<tr>
<th>Period</th>
<th>real-world</th>
<th>Risk-Neutral</th>
<th>Wang-T</th>
<th>Canonical</th>
<th>Tat</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K(5)$</td>
<td>0.93233</td>
<td>0.93577</td>
<td>0.96720</td>
<td>0.93219</td>
<td>0.93225</td>
</tr>
<tr>
<td>$K(10)$</td>
<td>0.83398</td>
<td>0.84430</td>
<td>0.90622</td>
<td>0.83378</td>
<td>0.83398</td>
</tr>
<tr>
<td>$K(15)$</td>
<td>0.69621</td>
<td>0.72295</td>
<td>0.80547</td>
<td>0.69648</td>
<td>0.69621</td>
</tr>
<tr>
<td>$K(20)$</td>
<td>0.51740</td>
<td>0.56046</td>
<td>0.65226</td>
<td>0.52053</td>
<td>0.51740</td>
</tr>
<tr>
<td>$K(25)$</td>
<td>0.31560</td>
<td>0.37075</td>
<td>0.44740</td>
<td>0.32598</td>
<td>0.31790</td>
</tr>
</tbody>
</table>
Conclusion

- Bayesian methods allows us to have prediction uncertainty in a systematic way via the prior distribution.

- The different choices of methods, produced "similar" results, except for the tatonnement approach. Under economic conditions, it shows that there really isn’t a need for a "premium" if both investor and hedger acts "rationally".

- Under the Wang transformed density, the premium is much higher than other methods. This is because the effect of the distortion operator causes a greater change in mortality directly compared with the risk-neutral method.
The End
Let $\psi = (\sigma^2, \Sigma, \theta)$. An MCMC method will be used to explore the posterior distribution and parameter states.

- Obtain initial draws denoted by $\psi^0$.
- Conditional on $\psi^0$, find the distribution of latent states via the Kalman Filter.
- Latent variable $\kappa_{1:T}$ drawn recursively from Backward Sampling (Carter and Kohn, 1994).
- Conditional on drawn latent variable, draw model parameters from their respective conditional posterior density.


